Method of fuzzy inference for one class of MISO-structure systems with non-singleton inputs

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Method of fuzzy inference for one class of MISO-structure systems with non-singleton inputs

V G Sinuk, M V Panchenko

Belgorod State Technological University, 46, Kostyukova St., Belgorod, 308012, Russia

E-mail: panchenko.maks@gmail.com

Abstract. In fuzzy modeling, the inputs of the simulated systems can receive both crisp values and non-Singleton. Computational complexity of fuzzy inference with fuzzy non-Singleton inputs corresponds to an exponential. This paper describes a new method of inference, based on the theorem of decomposition of a multidimensional fuzzy implication and a fuzzy truth value. This method is considered for fuzzy inputs and has a polynomial complexity, which makes it possible to use it for modeling large-dimensional MISO-structure systems.

1. Introduction

Presently, there are different methods of fuzzy inference. According to the classification, for example in [1], all methods can be divided into three types: logical, Mamdani-type inference and Takagi-Sugeno type inference. Most of them use crisp values as inputs of the simulated system. Well-known methods for fuzzy inputs are not applicable in most practical problems due to low computational efficiency, which does not allow implementing an inference for acceptable time in systems with a large number of inputs [2].

This article develops a logical type inference method for Non-Singleton in-puts, based on the decomposition theorem of fuzzy multidimensional implication. This theorem is proved under the condition that if the linguistic binding in the ante-cedent of fuzzy rules is modeled by the t-norm – min, or the t-conorm – max. The improvement of this method is due to the using of a fuzzy truth value, whose definition, for some membership functions, is performed analytically, and in the case of a piecewise linear representation of the membership function, an effective algorithm is developed [3].

2. Statement of the problem

The linguistic model is a set of fuzzy rules R_k, $k = \overline{1,N}$ in the form:

$$R_k\colon IF\ x_1\ is\ A_{1k} * x_2\ is\ A_{2k} * \ldots * \ x_n\ is\ A_{nk}\ THEN\ y\ is\ B_k, \tag{1}$$

where N – number of fuzzy rules; $A_{ik} \subseteq X_i$, $i = \overline{1,n}$, $B_k \subseteq Y$ – fuzzy sets, which are described by membership functions $\mu_{A_{ki}}(x_i)$ and $\mu_{B_k}(y)$, respectively; $[x_1, x_2, \ldots, x_n]$ – input variables of the linguistic model and $[x_1, x_2, \ldots, x_n]^T = \boldsymbol{x} \in X_1 \times X_2 \ldots \times X_n$. X_i, $i = \overline{1,N}$ and Y are the spaces of input and output variables. If $\boldsymbol{X} = X_1 \times X_2 \ldots \times X_n$ and $\boldsymbol{A_k} = A_{1k} \times A_{2k} \times \ldots \times A_{nk}$, then rule (1) is represented as a fuzzy implication:

$$R_k\colon \boldsymbol{A_k} \longrightarrow B_k, \qquad k = \overline{1,N}$$

Rule R_k can be formalized as a fuzzy relation, defined on the $\boldsymbol{X} \times Y$, $R_k \subseteq \boldsymbol{X} \times Y$, which is a fuzzy set with membership function:

$$\mu_{R_k}(\boldsymbol{x}, y) = \mu_{\boldsymbol{A_k} \longrightarrow B}(\boldsymbol{x}, y) = I(\mu_{A_k}(\boldsymbol{x}), \mu_B(y)),$$

where $I(\cdot)$ – implication, $*$ – linguistic bindings «AND» or «OR».

The task is to determine fuzzy output $B'_k \subseteq Y$ for the system represented in form (1), if there are fuzzy sets at inputs $\boldsymbol{A'} = A'_1 \times \ldots \times A'_n \subseteq \boldsymbol{X}$ or x_1 is A'_1 and … and x_n is A'_n.

According to the fuzzy modus ponens rule [4], fuzzy set B'_k is determined as a combination of $\boldsymbol{A'}$ and relation R_k:

$$B'_k = \boldsymbol{A'} \circ (\boldsymbol{A_k} \longrightarrow B_k) \quad (2)$$

The complexity of expression (2) is $O(|X|^{\mathrm{n}} * |Y|)$.

3. The method of inference based on the fuzzy truth value and the decomposition theorem

It is known that a particular case of the compositional rule of inference is the generalized modus ponens rule, which for systems with one input is described by the relation [1]:

$$\mu_{B'}(y) = \sup_{x \in X} \{\mu_{A'}(x) \overset{T}{*} I(\mu_A(\mathrm{x}), \mu_B(\mathrm{y}))\}, \quad (2)$$

where $\mu_{A'}(x)$, $\mu_A(x)$, $\mu_{B'}(y)$, $\mu_B(y)$ – membership functions, $\overset{T}{*}$ – t-norm, which is the intersection of fuzzy fact A' and fuzzy implication I, the argument of which is premise A and output B. Fuzzy sets are described in the space of reasoning X for premise and fact, and in Y - for value B and result of inference B'.

Using the rule of truth modification [5]:

$$\mu_{A'}(x) = \tau_{A/A'}(\mu_A(x)),$$

where $\tau_{A/A'}(\cdot)$ – fuzzy truth value of fuzzy set A about A', which represents compatibility membership function $CP(A, A')$ A towards A'; moreover A' is considered as reliable [6]:

$$\tau_{A/A'}(t) = \mu_{CP(A,A')}(t) = \sup_{\substack{\mu_A(x)=t \\ x \in X}} [\mu_A(x)], \ t \in [0,1]. \quad (3)$$

When passing from variable *x* to variable *t*, denoting $t = \mu_A(x)$:

$$\mu_{A'}(x) = \tau_{A/A'}(\mu_A(x)) = \tau_{A/A'}(t)$$

Then (3) is represented as:

$$\mu_{B'}(y) = \sup_{t \in [0,1]} \{\tau_{A/A'}(t) \overset{T}{*} I(t, \mu_B(y))\} \quad (4)$$

If membership functions $\mu_A(x)$ and $\mu_{A'}(x)$ are in Gaussian-shaped form:

$$f(x, a, b) = exp\left\{-\frac{(x-a)^2}{2b^2}\right\},$$

or in bell-shaped form:

$$f(x, a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}},$$

then, as follows from [3], fuzzy truth value (4) is determined analytically.

In the case of a piecewise linear representation of membership functions, an algorithm with polynomial complexity has been developed [3].

Let us consider the theorem of the decomposition of a multidimensional fuzzy implication [2]. This theorem can be used if linguistic binding «*AND*» in antecedent of rule (1) is modeling with *t-norm – min*, and linguistic binding «*OR*» is modeling with *t-conorm – max*.

In view of the above-mentioned facts, the decomposition theorem is formulated as follows:
if $I[\mu_{A_{ik}}(x_i), \mu_B(y)]$, $i = \overline{1,n}$ does not increase with respect to argument $\mu_{A_{ik}}(x_i)$, then in the case of a linguistic binding «*AND*»:

$$I\left(\mu_{\boldsymbol{A_k}}(\boldsymbol{x}), \mu_B(y)\right) = I\left(\underset{i=1,n}{min}\{\mu_{A_{ik}}(x_i)\}, \mu_B(y)\right) = \underset{i=1,n}{max}\left\{I\left(\mu_{A_{ik}}(x_i), \mu_B(y)\right)\right\};$$

in the case of a linguistic binding «*OR*»:

$$I\left(\mu_{\boldsymbol{A_k}}(\boldsymbol{x}), \mu_B(y)\right) = I\left(\underset{i=1,n}{max}\{\mu_{A_{ik}}(x_i)\}, \mu_B(y)\right) = \underset{i=1,n}{min}\left\{I\left(\mu_{A_{ik}}(x_i), \mu_B(y)\right)\right\}.$$

For a system with many inputs (3) has the form:

$$\mu_{B'}(y) = \underset{x \in \boldsymbol{X}}{sup}\{\mu_{\boldsymbol{A}'}(\boldsymbol{x}) \overset{T}{*} I(\boldsymbol{\mu}_{\boldsymbol{A_k}}(\boldsymbol{x}), \mu_B(y))\} \quad (5)$$

Under the conditions of the theorem of decomposition of a multidimensional fuzzy implication of non-increasing $I[\mu_{A_{ik}}(x_i), \mu_B(y)]$, $i = \overline{1,n}$ relatively $\mu_{A_{ik}}(x_i)$ and using linguistic binding «*AND*» (6) takes the form:

$$\mu_{B_k'}(y) = \max_{i=\overline{1,n}} \left\{\sup_{x_i \in X_i} \left\{\mu_{A_i'}(x_i) \overset{T}{*} I\left(\mu_{A_{ik}}(x_i),\ \mu_{B_k}(y)\right)\right\}\right\},\ k = \overline{1,N} \quad (6)$$

Expression (7) can be written in terms of fuzzy truth value, as follows from (4), and (7) will take the form:

$$\mu_{B_k'}(y) = \max_{i=\overline{1,n}} \left\{\sup_{t_i \in [0,1]} \left\{\tau_{A_{ik}/A_i}(t_i) \overset{T}{*} I\left(t_i, \mu_{B_k}(y)\right)\right\}\right\},\ k = \overline{1,N}. \quad (7)$$

Ratio (8) has polynomial computational complexity – $O(|t_i| * |Y| * n)$. The condition of non-increasing $I(t_i, \mu_{B_k}(y))$, $i = \overline{1,n}$ relatively t_i is performed for the following implications:
Lukasiewicz implication:

$$I\left(t_i, \mu_{B_k}(y)\right) = \min\left[1, 1 - t_i + \mu_{B_k}(y)\right];$$

Kleene-Dienes (binary) implication:

$$I\left(t_i, \mu_{B_k}(y)\right) = \max\left[1 - t_i, \mu_{B_k}(y)\right];$$

Reichenbach implication:

$$I\left(t_i, \mu_{B_k}(y)\right) = 1 - t_i + t_i * \mu_{B_k}(y);$$

Fodor implication:

$$I(t_i, \mu_{B_k}(y)) = \begin{cases} 1, & t_i \le \mu_{B_k}(y) \\ \max\left[1 - t_i, \mu_{B_k}(y)\right], & t_i > \mu_{B_k}(y) \end{cases}$$

Rescher implication:

$$I\left(t_i, \mu_{B_k}(y)\right) = \begin{cases} 1, & t_i \le \mu_{B_k}(y) \\ 0, & t_i > \mu_{B_k}(y) \end{cases};$$

Goguen implication:

$$I(t_i, \mu_{B_k}(y)) = \begin{cases} \min\left[1, \dfrac{\mu_{B_k}(y)}{t_i}\right], t_i > 0 \\ 1, \qquad\qquad\qquad t_i = 0 \end{cases};$$

Godel implication:

$$I\left(t_i, \mu_{B_k}(y)\right) = \begin{cases} 1, & t_i \le \mu_{B_k}(y) \\ \mu_{B_k}(y), & t_i > \mu_{B_k}(y) \end{cases};$$

Yager implication:

$$I\left(t_i, \mu_{B_k}(y)\right) = \begin{cases} 1, & t_i = 0 \\ [\mu_{B_k}(y)]^{t_i}, & t_i > 0 \end{cases};$$

Aliev implication (ALI-2):

$$I\left(t_i, \mu_{B_k}(y)\right) = \begin{cases} 1, & t_i \le \mu_{B_k}(y) \\ \min[(1 - t_i), \mu_{B_k}(y)], & t_i > \mu_{B_k}(y) \end{cases};$$

Aliev implication (ALI-3):

$$I\left(t_i, \mu_{B_k}(y)\right) = \begin{cases} 1, & t_i \le \mu_{B_k}(y) \\ \dfrac{\mu_{B_k}(y)}{\left[t_i + \left(1 - \mu_{B_k}(y)\right)\right]}, & t_i > \mu_{B_k}(y) \end{cases};$$

Aliev implication (ALI-4):

$$I\left(t_i, \mu_{B_k}(y)\right) = \begin{cases} \dfrac{1 - t_i + \mu_{B_k}(y)}{2}, & t_i > \mu_{B_k}(y) \\ 1, & t_i \le \mu_{B_k}(y) \end{cases}.$$

This property is shown graphically in tabulation $\mu_B(y)$ from 0 to 1 with step 0.1 for some implications (figure 1-4).

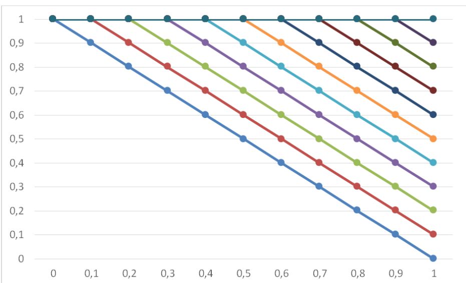

Figure 1. Lukasiewicz implication.

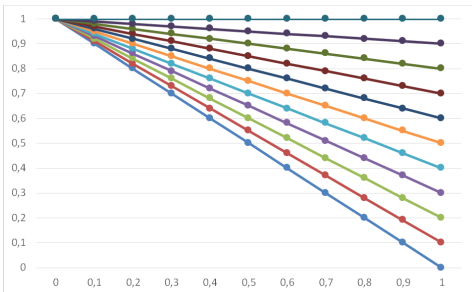

Figure 2. Reichenbach implication.

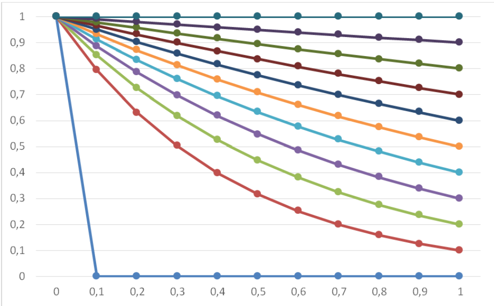

Figure 3. Yager implication.

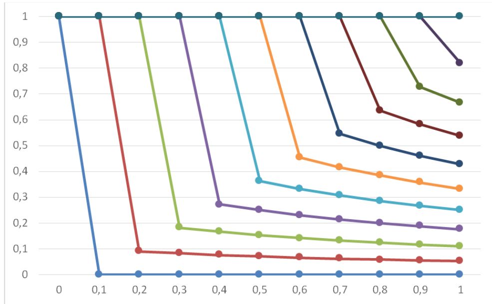

Figure 4. Aliev implication (ALI-3).

4. Inference of the output value for the rule block

Let us consider the output values for N rules (1), using the center of gravity method of defuzzification [1]:

$$\bar{y} = \frac{\sum_{k=\overline{1,N}} \bar{y}_k * \mu_{B'}(\bar{y}_k)}{\sum_{k=\overline{1,N}} \mu_{B'}(\bar{y}_k)}, \tag{8}$$

where $\bar{y}$ – crisp output in a system consisting of N rules of the form (1); $\bar{y}_k$, $k = \overline{1,N}$ – centers of membership functions $\mu_{B_k}(y)$, i.e. values in which $\max_y \mu_{B_k}(y) = 1$, B' is obtained as an input in logical-type systems, using the intersection operation [1] in accordance with expression:

$$B' = \bigcap_{j=\overline{1,N}} B'_j.$$

The membership function B' is calculated using the t-norm:

$$\mu_{B'}(y) = \underset{j=\overline{1,N}}{T} \mu_{B'_j}(y). \tag{9}$$

If implication $I(t_i, \mu_{B_k}(y))$, $i = \overline{1,n}$ is not increasing relatively t_i and linguistic binding «*AND*» is used in (1), then from (9), (10) and (8):

$$\bar{y} = \frac{\sum_{k=\overline{1,N}} \bar{y}_k \underset{j=\overline{1,N}}{T}\{\underset{i=\overline{1,n}}{max}\{\sup_{t_i\in[0,1]}\{\tau_{A_{ik}/A_i}(t_i) \overset{T}{*} I(t_i, \mu_{B_j}(\bar{y}_k))\}\}\}}{\sum_{k=\overline{1,N}} \underset{j=\overline{1,N}}{T}\{\underset{i=\overline{1,n}}{max}\{\sup_{t_i\in[0,1]}\{\tau_{A_{ik}/A_i}(t_i) \overset{T}{*} I(t_i, \mu_{B_j}(\bar{y}_k))\}\}\}}. \tag{10}$$

Relation (11) corresponds to the network structure (fig.5), where on the second level the following expression is determined:

$$F_{ijk}\{\cdot\} = \sup_{t_i\in[0,1]} \{\tau_{A_{i,k}/A_i}(t_i) \overset{T}{*} I(t_i, \mu_{B_j}(\bar{y}_k))\}$$

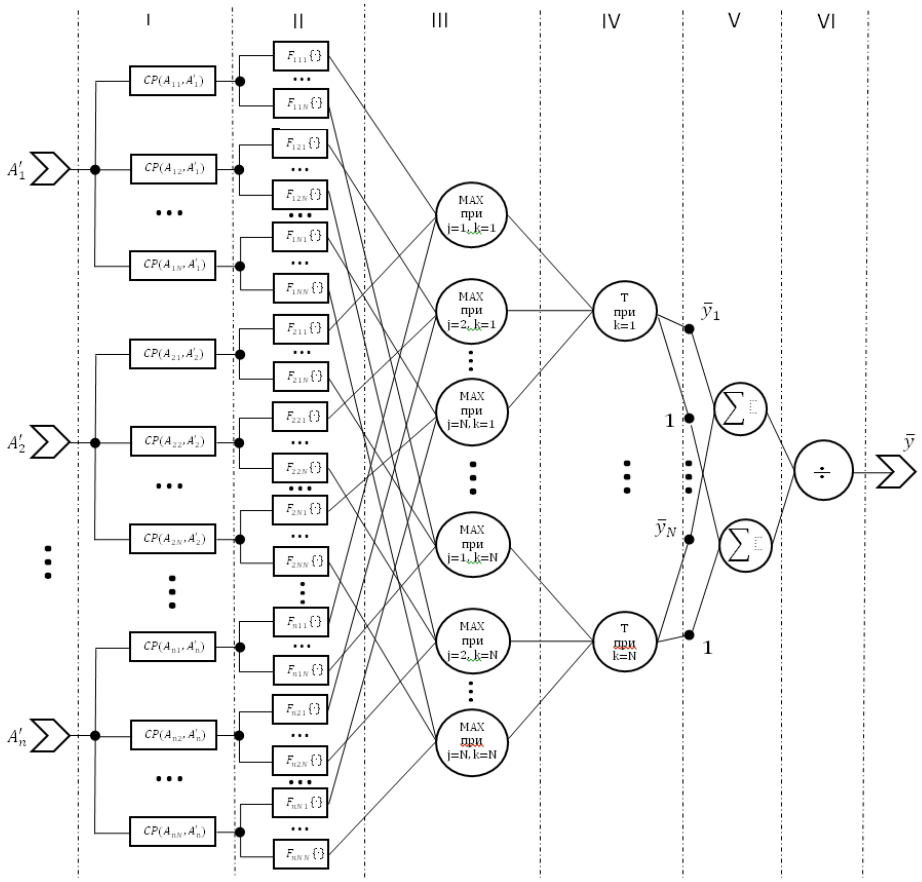

Figure 5. A network structure for the block of rules.

5. Conclusion

The paper proposes a method of fuzzy inference for systems of a logical type using the theorem of decomposition of a multidimensional fuzzy implication and fuzzy truth value. The computational complexity of the proposed method corresponds to polynomial algorithms.

The developed network structure for getting an output value for a rule block, makes it possible to conduct further studies on learning by varying the form of fuzzy implication, as well as the parameters of the membership functions, and, as follows from the network structure, to apply parallel computation methods.

Polynomial computational complexity makes it possible to use this method to solve problems of modeling systems with a large number of fuzzy inputs, such as diagnostics, prediction, and control [7].

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