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Relevance of deterministic chaos theory to studies in functioning of dynamical systems

S N Glagolev, S M Bukhonova, E D Chikina

Belgorod State Technological University named after V.G.Shoukhov, 46, Kostyukov St., Belgorod, 308012, Russia

E-mail: atchikina@mail.ru

Abstract The paper considers chaotic behavior of dynamical systems typical for social and economic processes. Approaches to analysis and evaluation of system development processes are studies from the point of view of controllability and determinateness. Explanations are given for necessity to apply non-standard mathematical tools to explain states of dynamical social and economic systems on the basis of fractal theory. Features of fractal structures, such as non-regularity, self-similarity, dimensionality and fractionality are considered.

1. Introduction

Humankind has long made a point of the fact that all the processes in the Universe exist as oscillations and there are no immovable objects. Process development is cyclic, i.e., it has a repetitive form, but does not always return the system back to its initial state. Such form of development is typical for dynamical systems of any nature: physical, chemical, biological, economic, technical, social, as well as of any configuration – complex, simple, conservative (closed or isolated), open, semi-open.

Let us note that no information enters a conservative system and no information leaves it. The solar system may serve as an example of a conservative system. As for semi-open systems, they are characterized by a feature of change in dynamic symmetry. The term *dynamic symmetry* was coined by Jay Hambidge, a scholar of architecture, to denote the principle of proportionality. Then, the term *dynamic symmetry* found its use in physics and other sciences to describe physical processes characterized by invariants. Being applied to dynamical systems which under action of external and internal forces undergo changes with time, this term characterizes regularity of natural morphogenesis, evolving with time.

Open and semi-open systems change their dynamic symmetry during movement, i.e., they evolve.

Semi-open systems are characterized with the process where information either enters the system, or leaves it. The result of such evolutionary process is quite simple, for there are only two variants of development: the system enters either the state of complete rest, or a state where absolutely everything changes.

In an open system, the information flows indirectly, through intermediates. In biological, chemical and physical systems they are molecules, in economic systems they are resources (capital, equipment and technologies, monetary funds, society, etc.). In social systems the intermediates in information transfer are further complemented with tastes, preferences, feelings, perceptions, etc., thus, when considering dynamical systems, it is necessary to keep in mind all the multifaceted nature of the information transfer.

2. Main part

To analyze and control behavior of complex dynamical systems, it is insufficient to use only such tools of dynamic symmetry as identification of regularities in development and growth, the idea of proportionality, numerical ratios, the Golden Section. The tools characterizing only the nature of the system itself are insufficient as well.

Formally, the dynamical systems are analyzed as mathematical models with the help of a diverse mathematical apparatus: theory of relations, systems of equations and inequalities describing regularities in such systems, linear and non-linear models, determinate and stochastic, static and dynamic, discrete and continuous, structural and functional (black box models).

However, the dynamical systems are so complex that they cannot be described with standard mathematical methods. So, for example, biological, physical, economic, social systems make the modeling process more complex and require application of non-standard approaches to their analysis and evaluation.

This paper considers some issues arising in social and economic systems.

Here one follows a definition of such class of dynamical systems as given in [1]: “a social and economic system is an integral aggregate of interlinked and interacting social and economic institutions (entities) and relationships in connection with distribution and consumption of material and immaterial resources, production, distribution, exchange and consumption of goods and services”

Thus, below, the social and economic system is considered as a complex, open, dynamical system, combining processes of production, distribution and consumption of material and immaterial assets.

The most important quality of any dynamical system, including social and economic ones, is stability, both structural and functional: conservation of structural and functional elements during a certain time interval. Besides, it is very important that structural and functional elements, integrating, regulating and coordinating the processes in the system adopt it to diverse influences of internal and external perturbations. Thus, stability of a dynamical system against influences appears as its inherent quality.

Efficiency of the system or its development depend on regulating parameters, rendering the system into a state of a more qualitative functioning.

It is worth noting that possibility of a dynamical system switching to a qualitative functioning phase may be analyzed with the catastrophe theory.

“Theory of irregularities and bifurcations provides a mathematical description of catastrophes, discontinuous variations, appearing as an unpredictable response of a system to smooth change in external conditions” [2].

The term *catastrophe theory* was first coined by René Frédéric Thom and Erik Christopher Zeeman.

The catastrophe theory identifies special cases of stable equilibrium, where small perturbation of the system leads to small deviation of the dynamical system from the equilibrium state.

In mechanics, equilibrium is such state of a system where the sum of all forces acting upon it is zero. The mechanical body (system) in the state of equilibrium is either resting, or moving in a uniform rectilinear fashion.

Equilibrium plays an important role in dynamical systems and may be applied to systems of any nature. Let us see several examples.

Chemical equilibrium is a state when a reaction of chemical processes proceeds to the same degree as a counter reaction; besides, there is no change in the number of components participating in the process.

Thermodynamic equilibrium is when processes are not changing (body temperature, pressure).

In game theory, equilibrium is seen as a stability of a player's strategy, as well as stability to behavior of entire coalitions (strong Nash equilibrium).

As for social and economic equilibrium, there is a vast plurality of opinions. The most universal understanding of equilibrium in a social and economic system is as follows: “an equilibrium is understood as consumer sovereignty, community of interests of different classes and social groups, as well as invariability or negligible variability of a certain economic situation" [1].

When modeling social and economic processes, an equilibrium is understood as mutual changeability of economic variables. Formally, for such systems the equilibrium is characterized with attaining a maximum or minimum value of a certain indicator.

There is also an opinion that equilibrium from the point of view of economics is a state, where perturbing factors and economic variables are balanced in such manner that the latter stay invariable. Economic categories of equilibrium are: "equilibrium price", "market equilibrium", "equilibrium quantity", etc.

For managerial decision-making, it is important to track a type of equilibrium. There are several: indifferent equilibrium, unstable, and stable equilibrium [3].

Unstable equilibrium is characterized by forces arising in small deviation from the equilibrium, working to increase the deviation; in indifferent equilibrium the system stays in equilibrium despite small deviations; stable equilibrium is characterized by forces, arising in small deviation from the equilibrium, working to reduce the deviation and put the system back into the equilibrium state.

Long-term stable equilibrium may be described with potential Lyapunov functions.

Let us note that applicability of such approach to modeling of social and economic systems is proven by many authors.

Speaking about using the catastrophe theory in this approach, one cannot leave unmentioned the bifurcation theory as well. The catastrophe theory is based upon this area of mathematics, which studies and classifies events of sudden changes in behavior of a system caused by small changes in its external environment.

Thus, the bifurcation theory analyzes the process where qualitative nature of solutions for a system of equations depends on parameters in this system.

The term *bifurcation* is quite established; it is formed from a Latin word *bifurcus* ("two-forked") and means different qualitative metamorphoses, restructuring or alterations in elements of a system when there are changes in their defining parameters.

This term was coined by Jules Henri Poincaré, a famous mathematician, physicist and mechanic, when he studied diverse appearance of the bifurcation process in the environment.

The concept of bifurcation, just as that of equilibrium, is important for analysis and evaluation of dynamical systems and is used to describe division (to divide - to bifurcate), as well as "division by two". For example, bifurcation of a river course, bifurcation of classes into two streams, bifurcation of dental roots, road bifurcation; in politics there are two directions, each one with its own development program, bifurcation of personality, branching of the social and economic system, bifurcation of trajectory, bifurcation of engagement, bifurcation of investment, etc.

Summing up, one may say that bifurcation of a social and economic system (SES) is an aggregate of all possible qualitative changes in SES behavior resulting from small-scale change in its parameters.

Another very important concept in studies of dynamical systems is a point of bifurcation. It is a critical moment, when a dynamical system transits from one systemic determinateness to another. Qualitative characteristics of the system after the transition undergo principal changes. Such points in SESs are called crises.

Changes in the functioning mode of a system are also called its points of bifurcation. If a dynamical system transits to an unpredictable mode, that is, in case a cascade of bifurcations appears (transition from a simple mode to a complex one with infinite doubling of period), a process of transition of the system into chaos occurs.

Thus, when the sequence of a system is described with two solutions, then with four, eight, sixteen, etc., such system transitions to a functioning mode with chaotic oscillations, requiring a constant doubling of the number of possible values.

It shall be noted that only non-linear dynamic systems transit to a chaotic oscillations state. Linear systems cannot be chaotic.

It is known that linear systems are described with linear differential equations, they provide the most accurate result and have a wide range of mathematical properties, they are used to understand the behavior of general dynamical systems.

Non-linear dynamic systems are correspondingly described with non-linear differential equations. Properties and characteristics of non-linear systems depend on the state of the system itself.

The social and economic system (SES) pertains to the systems of such type, it is complex, open, non-linear and functions in the environment of uncertainty and risk.

Unlike studies of linear dynamical systems, approaches to control of non-linear systems require extension of the mathematical apparatus.

Here let us highlight a key direction in studying non-linear dynamical systems – deterministic chaos theory.

There is no exact definition of the chaos theory in physics or mathematics, however, specialists in this object domain define chaos as instability, unpredictability, variability, non-linearity appearing in dynamical systems. Thus, the chaos theory studies behavior of non-linear dynamical systems subject to chaotic oscillations.

In the theory, there are concepts of *dynamic chaos* and *deterministic chaos*. These terms are synonymous and represent a phenomenon when a behavior of the non-linear dynamical system with determenistic laws appear random.

The foundation of the idea of deterministic (dynamic) chaos was formed in such areas of studies as: differential dynamics, theory of measure, functional analysis, singularity theory, topology, fractal analysis [4].

As noted above, the common apparatus of analytical calculations for individual trajectories of differential equations does not work in studies of non-linear dynamical system complex behavior. Thus, the main tasks of the deterministic chaos theory are studies of stability, fractal analysis of geometric structures, search for invariant measures and characteristics.

Let us note that in this case invariant measures are measures defined in the phase space of the system, invariant to evolution (changes, development) of the system itself with time. Such measures are applied, e.g., for averaging motion equations, in Lyapunov theories, in studies of fractal dimension of a system, etc.

Phase space of a dynamical system is also worth noting. Here the authors base on concepts and definitions adopted in physics and mathematics, so it is possible to consider phase space from the same point of view.

So, a phase space of a dynamical social and economic system is a space where a set of states of the system is represented in such way that there is a point in the phase space corresponding to each possible state of the social and economic system. The nature of the phase system of a dynamical system is such that the state of SES is represented in such space with a single point, while evolution of the system is represented with a movement of this point. To define phase space that shows behavior of the dynamical system as a whole, it is necessary to single out such phase coordinates in a phase plain that unequivocally defines the state of the system. Let us highlight that each phase coordinate reflects only a single state of the dynamical system (SES), with the exception for special points (in special points the vector field is zero, they are equilibrium points). Movement of this phase point will reflect changes in SES onto the phase plain in the phase space. Trace of movement of the phase point is a phase trajectory, the total of such trajectories forms the phase image of the system. Let us reiterate, that all the changes of the dynamical system through a period of time may be understood through its phase image.

A notion of an attractor is an important concept in the chaos theory. An attractor is a certain subset of the dynamical system phase subspace, which serves as a point of attraction for all the phase trajectories in a certain neighborhood when time approaches infinity. In other words, one may say that an attractor is a point of attraction of the dynamical system. A certain point or its trajectory may be an attractor of a system, or even a certain area with unstable trajectories, such as strange attractor. It is necessary to stress that if a system state is characterized by a strange attractor, it is impossible to determine behavior of its parts in all moments of time. Phase image of the strange attractor is a certain area, inside which random walks happen. Thus, modeling and prediction of behavior for a system in

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the stage of a strange attractor is significantly more complex.

Thus, unlike the normal attractor, it is the strange attractor which is interesting and unpredictable from the point of view of control. It is neither a curve nor a point. Geometry of the strange attractor is very complex, that is why there are issues arising with problem formulation in cases when it is in the strange attractor phase. It is deemed that the structure of the strange attractor is fractal, i.e., it is a set with irregular, unstable branched or peaked structure.

"*Fractal* is a term, coined by Benoit Mandelbrot from a Latin work meaning "broken". He noted that fractal is a geometric form, divided into components, with each component repeating the whole form at a smaller scale" [6]. Mandelbrot said that studying the complex dynamical systems is possible following two approaches: geometrically (visual geometry is like X-ray for a doctor) and analytically (exact mathematical apparatus, according to Mandelbrot, is analogous to medical analysis and instrument measurements). "A good doctor makes his diagnosis from both images and numbers. Scientists shall learn to do the same" [6].

By founding the fractal analysis theory, Mandelbrot displaced from the analytical toolkit such tools as: OLS, spectral analysis, sample theory, closed distributions and many other models and methods, used in studying the dynamical systems, SES in particular.

Fractals have certain properties, distinguishing them from other patterns. So, fractals are irregular, self-similar, dimensional and fractionality. Everything around us seeming unstable, random, irregular or unpredictable may be a fractal. Fractals of market, price fluctuation graphs, time series fractals – those are examples of fractals in economics. Fractals surround us everywhere: in the nature they are trees, leaves, coast lines, clouds; in architecture they are buildings, monuments, bridges, constructions; in medicine and biology they are lungs and other body parts, bacteria, viruses; in technology they are instruments, equipment, porous materials, petroleum chemistry. There are fractals in computer graphics, in designer models, in electrical engineering, in nanotechnologies, etc. By the way, a strange attractor in the phase space of deterministic chaos is a fractal as well. Thus, if the whole world is manifestation of fractal geometry, one shall know how to analyze fractals and apply them to practice.

When studying dynamical systems from the positions of the chaos theory, researchers juxtapose Euclidean geometry with Mandelbrot geometry. Euclidean geometry studies elementary geometry, the simplest properties of physical space. Curved elements in this geometry are smooth and locally straight, thus, they are always self-similar. As for Mandelbrot geometry, it studies fractal geometry where curves are infinitely peaked. A property of fractal geometry is that the fractal curve, at any magnification does not appear straight. Fractal geometry describes chaotic processes. Functions describing fractal curves are nondifferentiable, they have no derivatives, thus, it is impossible to determine a rate (speed) of change in such dynamical system.

Fractal analysis or fractal theory includes a number of tools, necessary for analyst to make managerial decisions. The tasks of the fractal analysis are to find the repeated structure in a dynamical system, to analyze and quantify them. Both concrete forms that repeat the system at a smaller scale, and abstract, statistical, probabilistic structures, like possibility of change in taste or fashion in a society, or probability of appearance of a new factor influencing SES, may be used. "Such structures may repeat upwards or downwards, the pattern may overlap or rotate (or both). The repetition may follow an exact deterministic rule or be coincidental" [6,7].

Fractal building is possible with any geometric objects, for example, squares, lines, triangles or spheres. The form, selected as a foundation of a fractal is called an initiator. Next, the process involves generators, which are patterns for building a fractal. The type of the twisting fractal curve depends on the initiator selected. An important part of fractal construction is the recursive rule; it determines the further plotting of a fractal curve. Thus, let us reiterate that complex dynamic systems, which are subject to chaotic oscillations, largely unpredictable, instable, are controlled with a specific theory with its own order, and non-linear models and tools – through the deterministic chaos theory.

3. Conclusions

Social and economic systems having non-linear, unpredictable (chaotic) nature, defy analysis and evaluation with standard approaches. Such systems require an informal, non-standard mathematical apparatus. Fractal theory, allowing studies of fractals with different dimensionality, analyzing influence of internal and external factors onto the system, allows predicting behavior of a dynamic system in the future, as well as identifying its unstable states. Fractal mathematics is quite complex and requires an analyst to have specific knowledge and skills; however, some fractal models may be used with a basic mathematical toolkit and electronic spreadsheets.

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References

[1] Dictionaries and Encyclopedias on Academica URL: http://www.krestonos.ru/battles/main/bitva_pri_askalone.html (Data extracted on: 13.06.2017).
[2] Arnold V I 1990 *Catastrophe Theory* (Moscow: Nauka) p 128
[3] Tradition. Free Russian Encyclopedia. URL: https://traditio.wiki (Date of retrieval: 13.06.2017).
[4] Doroshenko Yu A, Bukhonova C M, Somina I V 2015 Use and adjustment of r/s method for modeling, evaluation and prediction of investment attractiveness of an economic system on the foundations of chaos theory with application of fractal dimensionality. *Belgorod Economic Annals* **3** 3-9
[5] Mandelbrot, Benoit 2002 *The Fractal Geometry of Nature* (Moscow: Institute of Computer Research) p. 656
[6] Mandelbrot Benoit B, Hudson Richard L 2006 *The Misbehavior of Markets (Russian edition)* (Moscow: Williams) p. 400
[7] Mandelbrot Benoit B 2004 *Fractals and Chaos: The Mandelbrot Set and Beyond*(New York: Springer-Verland) p 308