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Precise method for calculating quantity of repetitive trials

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Annotation. The article analyzes a more precise method for calculating the quantity of repetitive trials required for an experimental research in the field of machine building. The work suggests, justifies and tabulates a new t_n -criterion of the minimal required number of repetitive trials. It is established that the new method is versatile, gives more stable and precise results, does not require a large number of repetitive trials, can be used for any samples, for any measurement accuracy requirements and assume calculation confidence probabilities.

1. Introduction

The required measurement precision and reliability of a set of repetitive (parallel) trials are known to be ensured by their identified minimal number. A number of works [1–5] describe the method to determine the quantity of trials, which implies a preliminary series of duplicated trials, statistical processing of the experimental results and determination of the minimal quantity of trials using the following equation:

$$n_{\min} \geq \left(\frac{\sigma \cdot t}{\bar{a} \cdot k_T} \right)^2, \quad (1)$$

where \bar{a} is the average value, and σ is the measurement mean square deviation determined by the results of preliminary trials; $[1]$ is required measurement accuracy in arbitrary units (in % or expressed as a decimal, as per problem statement); t is Student criterion from Table in [3] chosen depending on the number of trails n or number of degrees of freedom $f = n - 1$ and set confidence probability P_c .

If the required measurement precision is set in arbitrary units Δ (mm, s, N, MPa, etc.), then eq. (1) becomes:

$$n_{\min} \geq \left(\frac{\sigma \cdot t}{\Delta} \right)^2. \quad (2)$$

The analysis of eqs. (1) and (2) obviously shows their controversy. Logically, t and σ in the right part of the equations should strictly correspond to sought number n_{\min} in the left part. Since n_{\min} is unknown, then, evidently, the adoption of t and σ as per the results of a preliminary experiment with trial number n_{pre} is illogical, for generally $n_{\text{pre}} \neq n_{\min}$, which is confirmed by a lot of experiments. The authors have suggested a new method to calculate the minimum required number of repetitive trials. The present work describes the method.



2. Materials and Methods

The new method is based on both well-known points of statistical analysis of the measurements (samples) and the results of statistical analysis of numerous experimental studies of cutting tool durability performed at BSTU named after V.G. Shoukhov.

3. Results and Discussion.

Eqs. (1) and (2) are transformed so that one part of them contains only calculated parameters, while the other one - only table parameters. Since the table value of the Student t -criterion is determined from preset confidence probability and the number of degrees of freedom [6–10] $f = n - 1$, then n_{min} can be expressed as $n_{min} = f + 1$. Then eqs. (1) and (2) can be reduced to:

$$\frac{k_m \cdot \bar{a}}{\sigma} \geq \frac{t}{\sqrt{f+1}} \quad \text{and} \quad (3)$$

$$\frac{\Delta}{\sigma} \geq \frac{t}{\sqrt{f+1}}. \quad (4)$$

The expression in the right part of the equations contains only table interconnected parameters t and f taken from the table of Student coefficients and is the table criterion of the minimal number of repetitive trials. Let us denote it as $t_{rt} = \frac{t}{\sqrt{f+1}}$. The calculated criterion is the left parts of eqs. (3) and (4), i.e.:

$$t_{rc} = \frac{k_T \cdot \bar{a}}{\sigma} \quad \text{and} \quad (5)$$

$$t_{rc} = \frac{\Delta}{\sigma}. \quad (6)$$

Thus, to provide the preset precision and reliability of measurements, the following condition should be met:

$$t_{rc} = \frac{k_T \cdot \bar{a}}{\sigma} \geq t_{rt} = \frac{t}{\sqrt{f+1}} \quad \text{and} \quad (7)$$

$$t_{rc} = \frac{\Delta}{\sigma} \geq t_{rt} = \frac{t}{\sqrt{f+1}}. \quad (8)$$

Then, the minimal number of repetitive trials according to the new method is determined in the following consequence.

1. A preliminary experiment is conducted with n repetitive trials (3–4 trials are enough).
2. Then arithmetic mean deviation \bar{a} and mean square deviation σ of the measurements are calculated.
3. By known methods [1–5], rough errors (distinctly different measurements) are eliminated from the statistical set of measurements, and for the remaining set, \bar{a} and σ are recalculated.
4. Then the calculated value of the t_{rc} criterion of the minimal number of repetitive trials is determined using eqs. (5) or (6).
5. To meet requirements (7) and (8), the closest lowest value or that equal to the calculated table

value of the t_r criterion is chosen from the following extended table of the Student criterion, which for confidence probabilities $P_c = 0.90, 0.95$ and 0.99 is as follows.

Table 1. Table of t_{rep} criterion of minimal number of repetitive trials

f	$P_c = 0.90$		$P_c = 0.95$		$P_c = 0.99$	
	t	t_r	t	t_r	t	t_r
1	2	3	4	5	6	7
1	6.3130	4.4640	12.7060	8.9845	63.6560	45.0116
2	2.9200	1.6859	4.3020	2.4838	9.9240	5.7296
3	2.3534	1.1767	3.1820	1.5910	5.8400	2.9200
4	2.1318	0.9534	2.7760	1.2415	4.6040	2.0590
5	2.0150	0.8226	2.5700	1.0492	4.0321	1.6461
6	1.9430	0.7344	2.4460	0.9245	3.7070	1.4011
7	1.8946	0.6698	2.3646	0.8360	3.4995	1.2373
8	1.8596	0.6199	2.3060	0.7687	3.3554	1.1185
9	1.8331	0.5797	2.2622	0.7154	3.24498	1.0277
10	1.8125	0.5465	2.2281	0.6718	3.1693	0.9556
11	1.7950	0.5182	2.2010	0.6354	3.1050	0.8963
12	1.7823	0.4943	2.1788	0.6043	3.0845	0.8555
13	1.7709	0.4733	2.1604	0.5774	3.0123	0.8051
14	1.7613	0.4548	2.1448	0.5538	2.9760	0.7684
15	1.7530	0.4383	2.1314	0.5329	2.9467	0.7367
16	1.7450	0.4232	2.1190	0.5139	2.9200	0.7082
17	1.7396	0.4100	2.1098	0.4973	2.8982	0.6831
18	1.7341	0.3978	2.1009	0.4820	2.8784	0.6604
19	1.7291	0.3866	2.0930	0.4680	2.8609	0.6397
20	1.7247	0.3764	2.0860	0.4552	2.8453	0.6209
21	1.7200	0.3667	2.0790	0.4432	2.8310	0.6036
22	1.7167	0.3580	2.0739	0.4324	2.8188	0.5878
23	1.7139	0.3498	2.0687	0.4223	2.8073	0.5730
24	1.7109	0.3422	2.0639	0.4128	2.7969	0.5594
25	1.7081	0.3350	2.0595	0.4039	2.7874	0.5467
26	1.7050	0.3281	2.0560	0.3957	2.7780	0.5346
27	1.7033	0.3219	2.0518	0.3878	2.7707	0.5236
28	1.7011	0.3159	2.0484	0.3804	2.7633	0.5131
29	1.6991	0.3102	2.0452	0.3734	2.7564	0.5032
30	1.6973	0.3048	2.0423	0.3668	2.7500	0.4939
32	1.6930	0.2947	2.0360	0.3544	2.7380	0.4766
34	1.6909	0.2858	2.0322	0.3435	2.7284	0.4612
36	1.6883	0.2776	2.0281	0.3334	2.7195	0.4471

38	1.6860	0.2700	2.0244	0.3242	2.7116	0.4342
40	1.6839	0.2630	2.0211	0.3156	2.7045	0.4224
42	1.6820	0.2565	2.0180	0.3077	2.6980	0.4114
44	1.6802	0.2505	2.0154	0.3004	2.6923	0.4013
46	1.6787	0.2449	2.0129	0.2936	2.6870	0.3919
48	1.6772	0.2396	2.0106	0.2872	2.6822	0.3832
50	1.6759	0.2347	2.0086	0.2813	2.6778	0.3750
60	1.6706	0.2139	2.0003	0.2561	2.6603	0.3406
70	1.6689	0.1981	1.9944	0.2367	2.6479	0.3142
80	1.6640	0.1849	1.9900	0.2211	2.6380	0.2931
90	1.6620	0.1742	1.9867	0.2083	2.6316	0.2759
100	1.6602	0.1652	1.9840	0.1974	2.6259	0.2613

In line with the proposed method, the table can be further extended.

6. The chosen table t_r -criterion is used to choose the corresponding number of degrees of freedom f from the same table and then the minimal number of repetitive trials is calculated: $n_{\min} = f + 1$.

7. If the number of preliminary trials $n < n_{\min}$, then the experiment is continued until the total number of trials $n = n_{\min}$. Then \bar{a} and σ are recalculated. If $n \geq n_{\min}$, the preliminary experiment is considered to be sufficient and all preliminary calculations - to be trustworthy.

Example. After the experimental study of the durability of cutters and after elimination of rough errors, the following statistical durability sequence was obtained (min): 71.00; 66.00; 69.00; 72.00;

68.00; 67.00. The average durability is $\bar{T} = 68.83$ min. Let us determine the calculated value of the t_r -criterion of the minimal number of repetitive trials:

$$t_{rc} = \frac{K_T \bar{T}}{\sigma} = \frac{0.15 \cdot 68.83}{2.32} = 4.45$$

where $K_T = 0.15$ (15%) is the preset permissible measurement error.

From Table 1, the closest lowest value of the t_r -criterion is chosen taking into account the calculated figure of 4.45. With predetermined confidence probability $P_c = 0.95$, this value is 2.4838, which corresponds to the number of degrees of freedom $f = 2$ (see highlighted in Table 1). Then, the minimal required number of repetitive trials is $n_{\min} = f + 1 = 2 + 1 = 3$. Since in this example $n = 6 > n_{\min} = 3$, the number of trials should be considered as sufficient, and the results of the statistical processing of measurements should be considered as trustworthy.

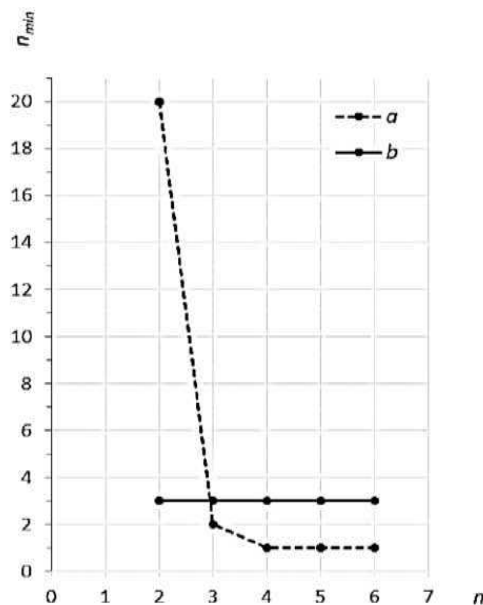
To compare the accuracy of the two methods — the traditional method based on inequalities (1) and (2) and the method based on the new t_r -criterion (see inequalities (3) and (4))—two corresponding calculation sets of the minimal number of repetitive trials were performed. In each of the sets, five alteration variants were processed using the initial statistical measurement sequence of the afore described example with the number of repetitive trials $n = 6, 5, 4, 3$ and 2. And starting from $n = 6$, each of the consequent variants of the statistical sequence was derived by rejecting the rightmost trial of the previous variant. The initial data and the calculation results are given in Table 2.

Table 2. Calculation of the minimal number of repetitive trials by two methods for five variants of experiments

	Parameters of statistical measurement sequences for five variants of experiments with the number of repetitive trials n				
	6	5	4	3	2
Experimental durability of tools [min]	71;66; 69;72; 68;67	71;66;69; 72;68	71;66;69; 72	71;66 ;69	71;66
Average experimental durability [min]	68.8	69.2	69.5	68.7	68.5
Measurement mean square deviation [min]	2.32	2.39	2.65	2.52	3.53
Calculated value of 4-criterion (t_{np})	4.45	4.34	3.93	4.09	2.91
Closest lowest table (t_n) value of t_n - criterion for $P_c=0.95$	2.48	2.48	2.48	2.48	2.48
Minimal required number of experiment repetitive trials obtained using:					
-traditional method as per eqs. (1) and (2)	1	1	1	2	20
- t_n -criterion	3	3	3	3	3

To clarify the difference between the two methods for determining minimal number n_{min} of repetitive trials, let us plot the alteration of n_{min} vs. alteration of the number of preliminary trials n : $n_{min}=f(n)$, using the calculation results from Table 2.

Numerous experiments in BSTU named after V.G. Shoukhov have established a large range of possible dependencies of n_{min} on the number of repetitive trials, average values of measurement dispersions, requirements for their accuracy and calculation confidence probability. Fig. 2. demonstrates several possible experimental plots with the varied number of preliminary trials of a single sample.

**Fig. 1.** Alteration of the minimal required number of repetitive trials n_{min} for variation of the number of preliminary trials n of the afore described experiment: a - traditional method for determining n_{min} ; b - t_n -criterion method

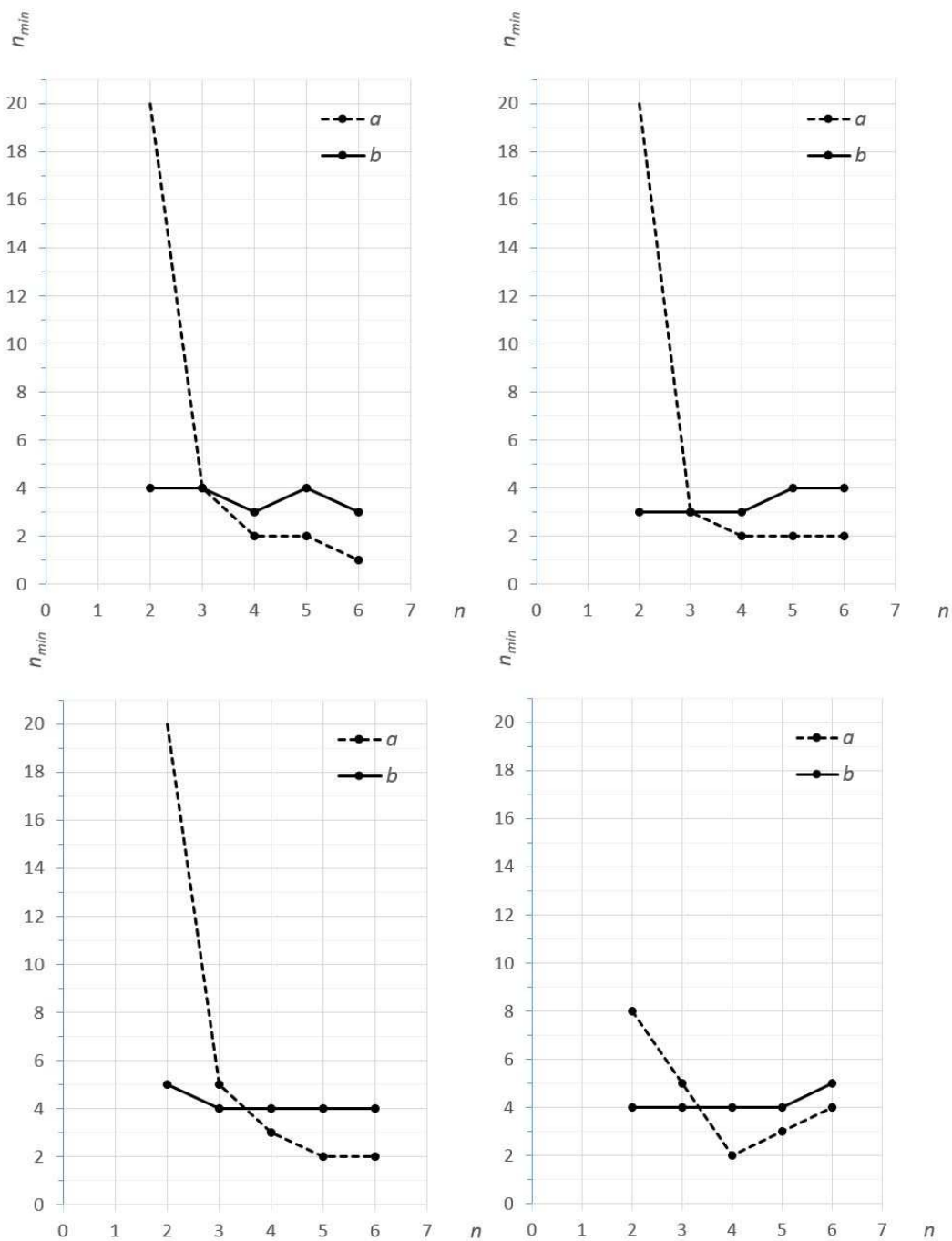


Fig. 2. Several variants of $n_{min}=f(n)$ plots obtained in real experiments.

4. Conclusions

Despite the diversity of $n_{min}=f(n)$ plots, the following conclusions can be made.

1. The new method for determining the minimal required number of repetitive trials using the t_n -criterion, unlike the traditional method, provides reliable results and does not need a lot of preliminary trials (3–4 are enough) and additional checking.
2. The traditional method for determining n_{min} with the number of preliminary trials $n > 3$ underestimates the value (sometimes at $n = 3$ as well).
3. The new method based on the t_n -criterion is second to none and can be used for any samples for any precision requirements and any assumed calculation confidence probability.

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