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# Research of movement process of fiber suspension in accelerating unit of wet grinding disintegrator

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**Abstract.** At the present stage of development of building material science, products reinforced with fibers of various origin (mineral, organic, metal and others) are commonly used. Determination of the optimal structure and the chemical composition of the fiber depends on a number of requirements for filler, binder, and other miscellaneous additives, etc. The rational combination of physical and chemical composition of the primary matrix of the product (e.g., binders, cement) with dispersion of anisotropic fiber of filler not only contributes to the strength of products, but also stabilizes their internal structure: prevents the occurrence of internal stress of the cement stone, increases the adhesive interaction of particles of cement at the contact boundary with fibers, etc.

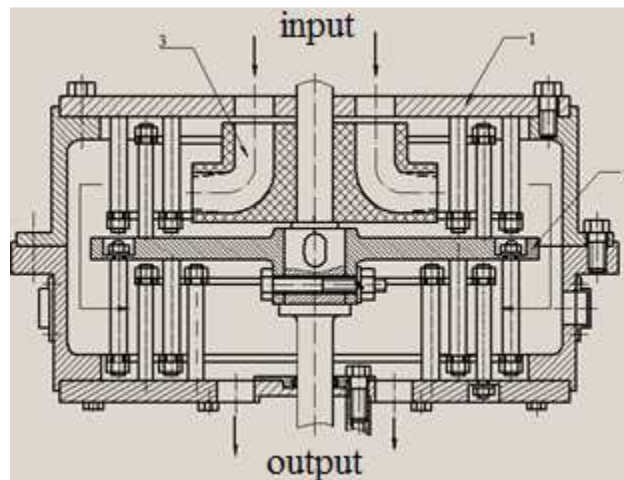
## 1. Introduction.

To improve the dispersion of the solid phase in the fibrous suspensions, wet grinding disintegrators are used. However, not all disintegrators can effectively disperse the fibers in a liquid medium. Technologically fibers must be processed at two stages. At the first stage, in the agitator mixer, pre-mixing and fiber fuzz are made. At the same time, it is necessary to take into account concentration of the solid phase in liquid relying on technological regulation of production of the reinforced products. Then, a final dispersing of fibers in a liquid medium is produced using rotor aggregates of integrated hydrodynamic [1]. The combination of various types of mechanical impacts on the fiber in the liquid contributes to the destruction of the core of the fiber in its both transverse and longitudinal cross-section.

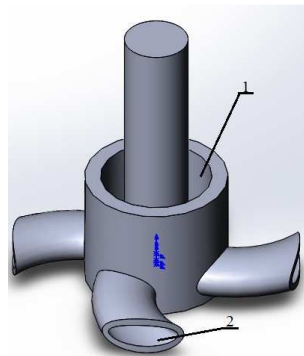
## 2. Main part.

There is such disintegrator construction [2, 3] in which addition of dispersion of fiber suspensions to the “dry” grinding process is possible. Fig. 1 shows the mill chamber of the disintegrator, which is equipped with an accelerating unit. This unit determines the productivity at the accelerating stage of fiber before its complex hydrodynamic processing. The 3D-model of this unit is shown in fig. 2





**Figure 1.** Construction of mill chamber of disintegrator equipped with accelerating unit:  
1 - outer rotor; 2 - inner rotor; 3 – accelerating unit.



**Figure2.** 3D-model of accelerating unit with toroidal-elliptic pipes mounted on shaft of disintegrator inner rotor: 1 - rotating annular channel; 2 - toroidal-elliptic channel.

Geometrical parameters of the accelerating unit and kinematic properties of the fiber suspension are related with basic equations of motion of a viscous incompressible liquid, which can be combined into a unified mathematical model which allows one to quantify the work of the accelerating unit. The description of the mathematical model of movement of fiber suspension in the accelerating unit of the disintegrator of wet grinding is as follows.

Let us assume that a viscous incompressible fluid - fiber suspension – is flowing vertically downwards in the annular rotating channel of the accelerating unit of the disintegrator as shown in Figure 1. Pre non-dispersed organic fibers, concentrated by volume of not more than 10%, were mixed in the agitator mixer with water.

To solve the problem, let us consider the stationary axisymmetric swirling flow of an incompressible viscous fluid that contains a small amount of impurity in the form of fibers. The movement of the suspension is carried out in accordance with Figure 2: at the entrance to the accelerating unit - in the rotating annular channel of the constant cross section - section 1; at the outlet of the accelerating unit - in a shrinking toroidal-elliptic channel - section 2.

Initially, direct-flow movement of the undispersed fiber slurry input to the rotating annular channel on the loading site in the disintegrator is an automodel distribution. In the rotating annular channel, the circumferential movement of the accelerating unit of the medium is superimposed on this circumferential movement of the medium, which is caused by the centrifugal forces. As a result, the trajectories of the motion of the particle of the water and the fibers in this section are helical lines.

It is clear that at the beginning of the rotating annular channel, there is a site of the primary alteration of motion of two-phase flow. Its length will be determined by the ratio of the intensities of convective and diffusion mechanisms of transfer in the peripheral speed. As soon as the distribution component of the circumferential velocity will conform to the law of rigid body rotation, this site will end. In view of the relatively small cross-section size of the flow and a modeling character of these calculations, the site of alteration movement of the suspension in the accelerating unit will not be considered.

For modeling the process of flowing of the fiber suspension in the vertical rotating annular channel of the accelerating unit, the stationary equations of isothermal axisymmetric subsonic flow of an incompressible fluid in a centrifugal field, which under the axisymmetric Navier - Stokes equations for an incompressible medium can be written as [4], are used:

$$u \frac{du}{dz} + v \frac{dv}{dr} = -\frac{1}{\rho} \frac{dp}{dz} + \nu \nabla^2 v; \quad (1)$$

$$u \frac{dv}{dz} + v \frac{dv}{dr} - \frac{w^2}{r} = -\frac{1}{\rho} \frac{dp}{dr} + \nu \left( \nabla^2 v - \frac{v}{r^2} \right); \quad (2)$$

$$u \frac{dw}{dz} + v \frac{dw}{dr} + \frac{v \cdot w}{r} = \nu \left( \nabla^2 w - \frac{w}{r^2} \right). \quad (3)$$

The flow continuity equation can be represented as:

$$\frac{du}{dz} + \frac{1}{r} \frac{d(v \cdot r)}{dr} = 0. \quad (4)$$

Here,  $z, r$  - the cylindrical coordinates;  $u, v, w$  - axial, radial and circumferential flow velocity in the selected coordinate system, m/s;  $\rho, p$  - density and pressure of the fiber suspension, respectively,  $\frac{kg}{m^3}$ , Pa;  $\nabla^2$  - plane Laplace operator defined from the equation:

$$\nabla^2 = \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r}. \quad (5)$$

Let us solve equation (1-4) in a sequence, using the method of substitution and easy self-similar solution of the problem (the entire distribution of which depends on the one variable coordinate) of the swirling flow of a viscous fluid in the annular channel. Do that using the equations:  $u(r) = U(r), v(r) = V(r), w(r) = W(\omega r)$ . This is denoted as:  $r_1$  - inner radius of the annular area that is equal to the diameter of the shaft of inner rotor  $r_i$ ;  $r_2$  - outer radius of the annular area. Then the flow pressure is:

$$p(z, r) = p_f(z) + \frac{\rho \omega^2}{2} (r_2^2 - r_1^2). \quad (6)$$

Here,  $\omega$  - angular velocity of the accelerating unit, rad/s;  $p_f(z)$  - linear function of pressure of  $z$ .

The strength of the internal friction of the flow that is acting on  $r_2$  in the direction of flow depends upon the axial flow velocity:

$$2\pi h_m \mu \left[ \left( r_1 \frac{dU(r)}{dr} \right)_{r_1+dr} - \left( r_1 \frac{dU(r)}{dr} \right)_{r_1} \right] = 2\pi h_m \mu \frac{d}{dr} \left( r_1 \frac{dU(r)}{dr} \right) dr, \quad (7)$$

where  $h_m$  - the height of the annular channel, m;  $U(r)$  - axial velocity of the suspension flow channel, m/s;  $dr(r)$  - change of the radius, m;  $\mu$  - the dynamic viscosity of the fiber suspension, Pa·s.

During stationary flow of the two-phase flow, the sum of the two forces is zero. Therefore, on the basis of Gauguin's assumptions and Poiseuille laws [4] the equation can be written:

$$\frac{d}{dr} \left( r_1 \frac{dU(r)}{dr} \right) = \frac{P_2 - P_1}{h_m \mu} r, \quad (8)$$

where  $P_2$  - the pressure of suspensions at the bottom (end) part of the annular space, MPa;

$P_1$  - the pressure of suspension in the upper (primary) part of the annular space, MPa.

The solution (8) vanishes when  $r = r_1$  and  $r = r_2$ ; therefore:

$$U(r) = \frac{P_2 - P_1}{4h_m \mu} \left[ (r_1^2 - r^2) + \frac{r_2^2 - r_1^2}{\ln\left(\frac{r_2}{r_1}\right)} \ln\left(\frac{r}{r_2}\right) \right]. \quad (9)$$

The radial component of the velocity does not depend on  $z$  and is directly proportional to coordinate  $r$ . Then from (1), let us find that:

$$V(r) \frac{dV(r)}{dr} - \nu \nabla^2 V(r) = 0. \quad (10)$$

The solution of equation (10) in the XY plane is reduced to the form (assuming that  $\frac{\partial^2}{\partial z} = 0$ ):

$$V(r) = 2\nu\nabla^2 r. \quad (11)$$

From equation (2.3), let us determine the value of the circumferential velocity with radius  $r$ :

$$W(r) = \sqrt{U(r)\omega \cdot r \frac{h_m}{(r_2-r_1)}}. \quad (12)$$

From expression (12), it is obvious that an increase of the height of the annular channel of accelerating unit circumferential velocity of undispersed fiber suspension is due to the increase of twisting flow.

With the increasing radius from  $r_1$  to  $r_2$  on the inner channel wall, a layer of two-phase flow will form. Growth of circumferential velocity of the suspension flow stops when its value equals the value of the circumferential speed of the inner wall of accelerating assembly. Adhesion will occur in the layer of the suspensions to the inner wall of the accelerating unit; processes of transportation and providing required acceleration to flow will not be effective. So, the height of the annular channel of the accelerating unit is an important constructive value that determines the effectiveness of its work.

Substituting equation (9) in (12) to determine the axial flow velocity, one will obtain:

$$W(r) = \sqrt{\frac{P_2-P_1}{4h_m\mu} \left[ (r_1^2 - r^2) + \frac{r_2^2-r_1^2}{\ln\left(\frac{r_2}{r_1}\right)} \ln\left(\frac{r}{r_2}\right) \right] \omega \cdot r \frac{h_m}{(r_2-r_1)}}. \quad (13)$$

It means that with increasing pressure of two-phase flow in a rotating annular channel, the value of the district forces increases: the flow intensively twists that confirms the functional significance of the accelerating unit and allows one to determine the value of its rational design parameters for stage 1. As result of modeling of the flow of the fiber suspension in a rotating annular vertical channel, epures of the flow velocity components were obtained for the three planes. The velocity profile of the fiber suspension in the accelerating unit is shown in Figure 3.

Resistance force  $F_s$  acts on the suspension in the flow on the suspension, which is proportional to the velocity of the transverse displacement:

$$F_s = 2 \frac{l}{d_s} m B_s \dot{r}, \quad (14)$$

where  $m$  - fiber weight, kg;  $l$  - fiber length, m;  $d_s$  - fiber diameter, m;  $\dot{r} = \frac{dr}{dt}$  - transverse coordinate of fiber at the center of gravity of mass, m;  $B_s$  - resistance coefficient of the fiber side surface in a rotating flow is determined by formula [4]:

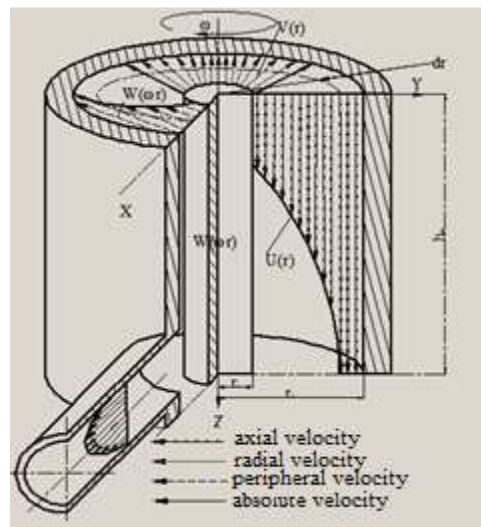
$$B_s = \frac{4\pi\mu}{l \cdot \ln(7,4Re)}, \quad (15)$$

here  $Re$  - Reynolds number is defined by the formula:

$$Re = \frac{\langle v \rangle d_s \rho}{\mu}, \quad (16)$$

where  $\langle v \rangle$  - absolute velocity suspension flow in m / s, which can be defined as:

$$\langle v \rangle = \sqrt{u^2(r) + v^2(r) + w^2(r)}. \quad (17)$$



**Figure 3.** Epures of the velocity of the fiber suspension in the booster unit disintegrator.

Due to the low concentration of fibers in the liquid phase, slidably individual fibers as they move in the longitudinal direction (along the axis OZ) are neglected. Let us assume that in this direction they are moving with the speed equal to the absolute flow velocity determined from equation (17). Then the basic equation of the dynamics of fibers movement in a radial direction, which takes into account only the centrifugal force of inertia and resistance of the medium, can be written as:

$$m \cdot \frac{d^2r}{dt^2} = m \frac{U_s^2}{r} - 2B_s \frac{dr}{dt}. \quad (18)$$

Here  $U_s$  - centrifugal speed of fiber m / s, determined as:

$$U_s = \omega_s r, \quad (19)$$

where  $\omega_s$  - the angular velocity of rotation oriented fibers in the annular space, rad / s;  $r$  - arbitrary radius of gyration of the fiber in the annular channel, m.

Using (18) and expression (19), while reducing (18) by the mass fibers, let us obtain:

$$\frac{d^2r}{dt^2} = \omega_s^2 r - 2B_s \frac{dr}{dt}. \quad (20)$$

Analytical solution (20) has the form:

$$r = e^{\gamma t}. \quad (21)$$

Substituting (21) into (20), one gets:

$$\gamma^2 e^{\gamma t} + 2\gamma e^{\gamma t} - \omega_s^2 e^{\gamma t} = 0. \quad (22)$$

An algebraic solution of equation (22) will be real numbers:

$$\gamma_{1,2} = -B_s \pm \sqrt{B_s^2 + \omega_s^2}. \quad (23)$$

The general solution of equation (22) will be presented as the following distribution:

$$r(t) = C_1 e^{\gamma_1 t} + C_2 e^{\gamma_2 t}. \quad (24)$$

where  $C_{1,2}$  - integration constants determined from initial conditions:

$$t = 0; \quad r = r_0; \quad \frac{dr}{dt} = 0.$$

Thus, from (24) with initial conditions:

$$\begin{cases} C_1 \gamma_1 + C_2 \gamma_2 = 0, \\ C_1 + C_2 = r_0. \end{cases} \quad (25)$$

Substituting (26) to (28) defines constant integration constants:

$$\begin{cases} C_1 = \frac{r_0}{2} \cdot \frac{B_s + \sqrt{B_s^2 + \omega_s^2}}{\sqrt{B_s^2 + \omega_s^2}}, \\ C_2 = -\frac{r_0}{2} \cdot \frac{B_s - \sqrt{B_s^2 + \omega_s^2}}{\sqrt{B_s^2 + \omega_s^2}}. \end{cases} \quad (26)$$

Based on the analytical studies from (18) to (26), let us obtain an equation for determining the radial velocity of the fiber suspensions in a rotating flow at any radius of the annular channel:

$$V_V = \gamma_1 C_1 e^{\gamma_1 t} + \gamma_2 C_2 e^{\gamma_2 t}. \quad (27)$$

After substituting all components of the equation in (30), let us finally get:

$$V_V = \frac{r_0}{2} \left[ (B_s - \sqrt{B_s^2 + \omega_s^2}) \cdot \frac{B_s - \sqrt{B_s^2 + \omega_s^2}}{\sqrt{B_s^2 + \omega_s^2}} e^{(-B_s - \sqrt{B_s^2 + \omega_s^2})t} - (B_s + \sqrt{B_s^2 + \omega_s^2}) \cdot \frac{B_s + \sqrt{B_s^2 + \omega_s^2}}{\sqrt{B_s^2 + \omega_s^2}} e^{(-B_s + \sqrt{B_s^2 + \omega_s^2})t} \right]. \quad (28)$$

The obtained expression for the radial velocity of the fibers in the liquid phase determines the relationship between the kinematic characteristics of suspensions, and structural and technological parameters of the accelerating unit. Due to increasing Re of the velocity of the two-phase flow and fibers in the radial direction, the turbulent mixing is observed due to expression (15) for reducing the coefficient of resistance  $B_s$  of fiber in a rotating flow.

On the basis of the structural features of the accelerating unit and considering equation (9) for the movement of a viscous fluid in a vertical annular channel, one can write an expression for determination of the fluid flow rate:

$$Q_y = \frac{\pi \rho \cdot (P_2 - P_1)}{8 \mu h_m} \left[ (r_2^4 - r_1^4) - \frac{r_2^2 - r_1^2}{\ln\left(\frac{r_2}{r_1}\right)} \right]. \quad (29)$$

The average rate of suspension in the pipe of the annular cross-section is equal to:

$$\bar{v} = \frac{1}{2} v_0, \quad (30)$$

where  $v_0$  - maximum speed in the middle section of the annular channel, m/s.

Then taking into account (30), expression (29) takes the form:

$$Q_y = \pi \rho \frac{v_0}{2} \left[ (r_2^4 - r_1^4) - \frac{r_2^2 - r_1^2}{\ln\left(\frac{r_2}{r_1}\right)} \right]. \quad (31)$$

Because the values of  $r_1$  and  $r_2$  are of the same order and differ slightly from each other by influence of centrifugal force, the suspension movement in the vertical pipe of the accelerating unit is neglected. To determine the speed and consumable parameters of the movement of two-phase flow in the accelerating unit at the section of the toroidal-elliptic pipes, it is necessary to consider that:

$$Q_y \geq n_3 Q_3, \quad (32)$$

where  $Q_y$  - suspension flow rate at the vertical annular section, kg/s;  $n_3$  - number of toroidal-elliptic pipes, pieces;  $Q_3$  - suspension flow rate through the toroidal-elliptic pipe, kg/s.

With a probable method in [5], [6], [7] for determining velocity  $v$ , let us select the equation and factors in this solution so as to satisfy the boundary condition at the pipe wall:  $v = 0$ . Let us direct axes Y and Z along the principal normal axes of the cross-section of the toroidal-elliptic tube (Figure 3), and obtain a solution in the form:

$$v = AY^2 + BZ^2 + v_0, \quad (33)$$

where  $A$  and  $B$  - geometric coefficients.

Expression (36) satisfies the equation:

$$2A + 2B = -\frac{P_2 - P_1}{h_m \mu}. \quad (34)$$

On the inner surface of the toroidal-elliptic pipe,  $v = 0$ ; then:



$$AY^2 + BZ^2 + v_0 = 0. \quad (35)$$

Let us reduce (35) to the equation of the elliptical cross-section pipe:

$$\frac{Y^2}{a^2} + \frac{Z^2}{b^2} - 1 = 0. \quad (36)$$

Then the coefficients can be represented and calculated as:

$$A = -\frac{v_0}{a^2}; \quad B = -\frac{v_0}{b^2}. \quad (37-38)$$

Solving consistently equations (36-38), let us obtain an expression for determining the flow rate of the viscous fluid in the pipe axis of the elliptical cross section:

$$v_0 = \frac{P_2 - P_1}{2h_m \mu} \cdot \frac{a^2 b^2}{(a^2 + b^2)}. \quad (39)$$

Thus, expression (39) on the basis of (36) takes form:

$$v = v_0 \cdot \left(1 - \frac{Y^2}{a^2} - \frac{Z^2}{b^2}\right). \quad (40)$$

Analyzing (43), one can argue that there is an increase in geometric parameters a and b of the elliptic cross section of the accelerating unit pipes speed of the undispersed fiber suspension by centrifugal force, and therefore it can be argued (6) that the pressure of the suspension increases too.

Now let us determine the flow rate of suspension through the pipe of the elliptical cross-section. The surfaces on which speed v is constant - elliptical cylinders:

$$\frac{Y^2}{a^2} + \frac{Z^2}{b^2} = 1. \quad (41)$$

Half-axes are determined by the formulas:

$$a'^2 = a^2 \frac{v_0 - v}{v_0}; \quad b'^2 = b^2 \frac{v_0 - v}{v_0}. \quad (42)$$

Consequently, the fiber suspension flow rate through one pipe of an elliptic cross section is determined from the expression:

$$Q_3 = \rho \int v \cdot dS = -\rho \frac{\pi ab}{v_0} \int_{v_0}^0 v dv. \quad (43)$$

After integrating (43), one can finally obtain:

$$Q_3 = \frac{\pi \rho ab}{2} v_0. \quad (44)$$

Expression (44) is a particular case for the calculation of the flow rate of the toroidal-elliptic pipe, and is obtained by integrating the equations of motion of the suspension inside the accelerating unit disintegrator [8-10].

### 3. Conclusion

Thus, the mathematical model that describes the motion of the fiber suspension in the accelerating unit of the wet grinding disintegrator is obtained. The model describes the nature of the suspension of movement in all areas of the accelerating unit, depending on its design and technological features.

### 4. Acknowledgments

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